



Three Flavor QCD at High Temperatures

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We have continued our study of the phase diagram of high temperature QCD with three flavors of improved staggered quarks. We are performing simulations with three degenerate quarks with masses less than or equal to the strange quark mass m_s and with degenerate up and down quarks with masses $m_{u,d}$ less than the strange quark mass. For the quark masses studied to date, we find a crossover that strengthens as $m_{u,d}$ decreases, rather than a *bona fide* phase transition. We present new results for the crossover temperature extrapolated to the physical value of $m_{u,d}$, and for quark number susceptibilities.

1. THE PHASE DIAGRAM

The MILC Collaboration is studying high temperature QCD with three flavors of improved staggered quarks [1] using the Asqtad quark action [2]. Simulations are being carried out with lattice spacings $1/4T$, $1/6T$ and $1/8T$. We are considering two cases: 1) all three quarks have the same mass m_q ; and 2) the two lightest quarks have equal mass $m_{u,d}$, while the mass of the third quark is fixed at approximately that of the strange quark m_s . We refer to these cases as $N_f = 3$ and $N_f = 2 + 1$, respectively. We have carried out studies with $0.2m_s \leq m_q \leq m_s$ for the $N_f = 3$ case, and with $0.1m_s \leq m_{u,d} \leq m_s$ for $N_f = 2 + 1$. At the masses we have studied to date, we find rapid crossovers, which sharpen as the quark mass is reduced, rather than a *bona fide* phase transition. During the past year we have extended and clarified our calculations of

the quark number susceptibilities, which provide an excellent signal for the crossover, and which are directly related to event by event fluctuations in heavy ion collisions. Comparison of susceptibilities at different lattice spacings show that our results are close to the continuum values. We have also sharpened our estimates of the crossover temperature for both the $N_f = 3$ and $N_f = 2 + 1$ cases.

For both the $N_f = 3$ and $2 + 1$ cases we have carried out thermodynamics studies on lattices with four, six and eight times slices, and aspect ratio $N_s/N_t = 2$. Here N_s and N_t are the spatial and temporal dimensions of the lattice in units of the lattice spacing. At the lightest quark masses, we also performed some simulations with aspect ratio three, and obtained results that are indistinguishable from those with aspect ratio two. For both cases the standard thermodynamic quantities show a crossover from confined behavior at low temperature to deconfined behavior at high

*Presented by R.L. Sugar

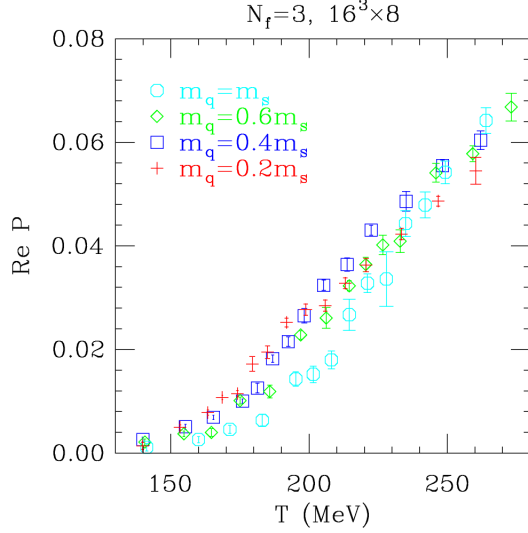


Figure 1. The real part of the Polyakov loop on $16^3 \times 8$ lattices for $N_f = 3$.

temperature, as is illustrated in Fig. 1 where we plot the real part of the Polyakov loop for $N_f = 3$ on $16^3 \times 8$ lattices. There is a slight trend for the temperature dependence of the Polyakov loop to be steeper for larger quark masses. This is to be expected, since at sufficiently large quark masses, it is a *bona fide* order parameter. The $\bar{\psi}\psi$ susceptibility χ_{tot} provides a clear signal for the crossover. It is given by

$$\chi_{\text{tot}} = \frac{\partial}{\partial m} \langle \bar{\psi}\psi \rangle. \quad (1)$$

We plot this quantity as a function of temperature on $8^3 \times 4$ lattices in Fig. 2. Note the increase in the height of the peak as the quark mass is decreased. Our work and that of the Bielefeld group [3] strongly suggests that in the real world, $N_f = 2 + 1$, there is no phase transition at the physical quark masses. With this assumption, we have estimated the critical temperature for $N_f = 2 + 1$ at the physical value of $m_{u,d}$ through an extrapolation of the form

$$r_1 T_c = c_0 + c_1 (m_\pi/m_\rho)^d + c_2 (aT_c)^2, \quad (2)$$

where we evaluated T_c for each value of N_t and m_π/m_ρ for which we have made measurements

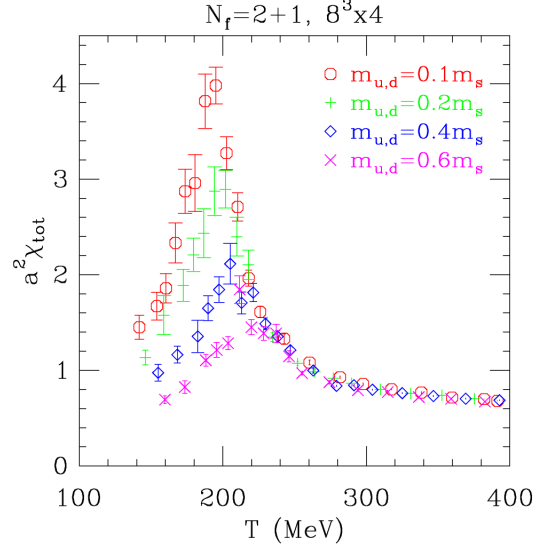


Figure 2. The $\bar{\psi}\psi$ susceptibility as a function of temperature on $8^3 \times 4$ lattices for $N_f = 2 + 1$.

from the peak in the $\bar{\psi}\psi$ susceptibility. For a second order phase transition in the $O(4)$ universality class at $m_{u,d} = 0$, $d = 2/\beta\delta \approx 1.08$. We find that $T_c = 169(12)(4)$ MeV with a χ^2 of 2.1 for 11 degrees of freedom. The first error is the fit error, the second from the uncertainty in r_1 , taken as $0.317(7)$ fm [4]. To test the sensitivity of T_c to d , we have also performed a fit with $d = 2$, which yields $T_c = 174(11)(4)$ MeV with a χ^2 of 1.5 for 11 degrees of freedom. So, the goodness of the fit does not allow us to prefer either of them.

2. QUARK NUMBER SUSCEPTIBILITIES

In order to study the quark number susceptibilities [5,6], we introduce chemical potentials μ_α coupled to a set of mutually commuting conserved charges Q_α . The quark number susceptibilities are related to event-by-event fluctuations in heavy ion collisions [7] by the fluctuation-dissipation theorem

$$\chi_{\alpha,\beta}(T) = \langle (Q_\alpha - \langle Q_\alpha \rangle)(Q_\beta - \langle Q_\beta \rangle) \rangle. \quad (3)$$

We work at $\mu_\alpha = 0$, so the brackets, $\langle \rangle$, in Eq. (3) indicate averages weighted by the standard, real Euclidean action for QCD, and $\langle Q_\alpha \rangle = 0$.



Rather than choosing the three independent charges to be the number operators for up, down and strange quarks, it appears more physical to take them to be the z -component of isospin, Q_I , the hypercharge, Q_Y , and the baryon number, Q_B . The rows and columns of the susceptibility matrix, $\chi_{\alpha,\beta}(T)$ are then labeled by I , Y and B . In the $N_f = 3$ case, where $m_u = m_d = m_s$, χ is a diagonal matrix, and there are no correlations among fluctuations in Q_I , Q_Y and Q_B , while for the $N_f = 2 + 1$ case, where $m_u = m_d$, the only correlations are between fluctuations in hypercharge and baryon number. For temperatures below the phase transition or crossover, the lightest particle that can be excited by a chemical potential coupled to Q_I is the pion, while for chemical potentials coupled to Q_Y and Q_B it is the kaon and the nucleon, respectively. Above the transition temperature each of the chemical potentials can excite quark states that are much lighter than hadrons, so we expect the diagonal elements of χ to increase sharply in the vicinity of the transition, and they do. This is illustrated in Fig. 3 where we plot the diagonal elements of the susceptibility matrix, $\chi_{I,I}$, $\chi_{Y,Y}$ and $\chi_{B,B}$, as a function of temperature for two light quarks with mass $0.2 m_s$ and one heavy quark with mass m_s on $12^3 \times 6$ lattices. $\chi_{Y,Y}$ and $\chi_{B,B}$ have been multiplied by factors of $3/4$ and $3/2$ respectively, so that the quantities plotted approach the same high temperature limit as $\chi_{I,I}$. Also shown is $\chi_{Y,B}$, the only non-zero off-diagonal matrix element of χ for $m_u = m_d$. It measures correlations between fluctuations in the hypercharge and baryon number. The coefficient of $\chi_{Y,B}$ in this figure is the geometric mean of those for $\chi_{Y,Y}$ and $\chi_{B,B}$. The good agreement between these results, and similar ones on lattices with eight time slices, indicate that they are close to their physical values.

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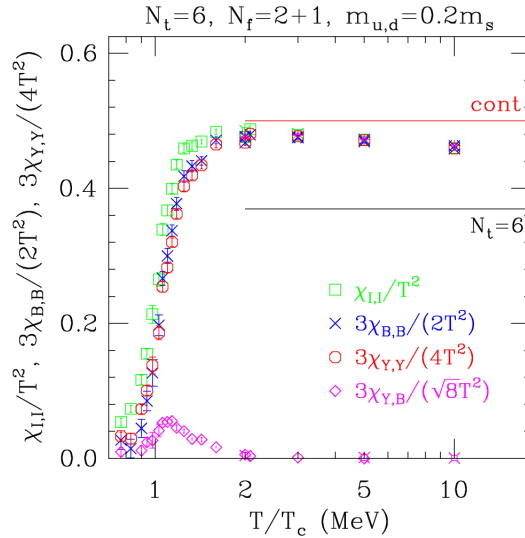


Figure 3. The non-vanishing elements of the susceptibility matrix for two light quarks of mass $0.2 m_s$ and one heavy quark of mass m_s on $12^3 \times 6$ lattices.

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